1. Dada la expresión regular $\mathbf{E} = (\mathbf{ab^*}\ +\ \mathbf{a^*c})^*\,\mathbf{b}$. Construye un autómata finito determinista que la reconozca empleando el método de Brzozowski.

$$
\begin{aligned}
\mathbf{a^{-1}[E]} &= a^{-1}[(ab^*+a^*c)^*]b+\delta((ab^*+a^*c)^*)a^{-1}[b] = \\
&= a^{-1}[ab^*+a^*c](ab^*+a^*c)^*b+1\cdot 0 = \\
&= \left(a^{-1}[ab^*]+a^{-1}[a^*c]\right)(ab^*+a^*c)^*b = \boxed{\mathbf{(b^*+a^*c)(ab^*+a^*c)^*b}}
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{b^{-1}[E]} &= b^{-1}[(ab^*+a^*c)^*]b+\delta((ab^*+a^*c)^*)b^{-1}[b] = \\
&= 0\cdot b+1\cdot 1 = \boxed{\varepsilon}
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{c^{-1}[E]} &= c^{-1}[(ab^*+a^*c)^*]b+\delta((ab^*+a^*c)^*)c^{-1}[b] = \\
&= c^{-1}[ab^*+a^*c](ab^*+a^*c)^*b+1\cdot 0 = \\
&= \left(c^{-1}[ab^*]+c^{-1}[a^*c]\right)(ab^*+a^*c)^*b = \\
&= (0+1)(ab^*+a^*c)^*b = \boxed{\mathbf{(ab^*+a^*c)^*b = E}}
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{a^{-1}[a^{-1}[E]]} &= a^{-1}[(b^*+a^*c)(ab^*+a^*c)^*b] = \\
&= a^{-1}[(b^*+a^*c)](ab^*+a^*c)^*b+\delta((b^*+a^*c))a^{-1}[(ab^*+a^*c)^*b] = \\
&= (a^{-1}[b^*]+a^{-1}[a^*c])(ab^*+a^*c)^*b+1\cdot a^{-1}[E] = \\
&= (0+a^*c)(ab^*+a^*c)^*b+(b^*+a^*c)(ab^*+a^*c)^*b = \\
&= \boxed{\mathbf{(a^*c)(ab^*+a^*c)^*b+(b^*+a^*c)(ab^*+a^*c)^*b}} \equiv \mathbf{a^{-1}[E]}
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{b^{-1}[a^{-1}[E]]} &= b^{-1}[(b^*+a^*c)(ab^*+a^*c)^*b] = \\
&= b^{-1}[(b^*+a^*c)](ab^*+a^*c)^*b+\delta((b^*+a^*c))b^{-1}[(ab^*+a^*c)^*b] = \\
&= (b^{-1}[b^*]+b^{-1}[a^*c])(ab^*+a^*c)^*b+1\cdot b^{-1}[E] = \\
&= (b^*+0)(ab^*+a^*c)^*b+1\cdot\varepsilon = \boxed{\mathbf{b^*(ab^*+a^*c)^*b}+\varepsilon}
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{c^{-1}[a^{-1}[E]]} &= c^{-1}[(b^*+a^*c)(ab^*+a^*c)^*b] = \\
&= c^{-1}[(b^*+a^*c)](ab^*+a^*c)^*b+\delta((b^*+a^*c))c^{-1}[(ab^*+a^*c)^*b] = \\
&= (c^{-1}[b^*]+c^{-1}[a^*c])(ab^*+a^*c)^*b+1\cdot c^{-1}[E] = (0+1)E+E = \boxed{\mathbf{E}}
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{a^{-1}[b^{-1}[a^{-1}[E]]]} &= a^{-1}[b^*(ab^*+a^*c)^*b+\varepsilon] = \\
&= a^{-1}[b^*(ab^*+a^*c)^*b]+a^{-1}[\varepsilon] = a^{-1}[b^*]E+\delta(b^*)a^{-1}[E]+0 = \\
&= 0\cdot E+1\cdot a^{-1}[E] = \boxed{\mathbf{a^{-1}[E]}}
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{b^{-1}[b^{-1}[a^{-1}[E]]]} &= b^{-1}[b^*(ab^*+a^*c)^*b+\varepsilon] = \\
&= b^{-1}[b^*(ab^*+a^*c)^*b]+b^{-1}[\varepsilon] = b^{-1}[b^*]E+\delta(b^*)b^{-1}[E]+0 = \\
&= b^*E+1\cdot b^{-1}[E] = b^*E+1\cdot\varepsilon = \boxed{\mathbf{b^*(ab^*+a^*c)^*b}+\varepsilon} \equiv \mathbf{b^{-1}[a^{-1}[E]]}
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{c^{-1}[b^{-1}[a^{-1}[E]]]} &= c^{-1}[b^*(ab^*+a^*c)^*b+\varepsilon] = \\
&= c^{-1}[b^*(ab^*+a^*c)^*b]+c^{-1}[\varepsilon] = c^{-1}[b^*]E+\delta(b^*)c^{-1}[E]+0 = \\
&= 0\cdot E+1\cdot c^{-1}[E] = 0+E = \boxed{\mathbf{E}}
\end{aligned}
$$

Estados finales: $\mathbf{b^{-1}[E]}$, $\mathbf{b^{-1}[a^{-1}[E]]}$

2. Dadas las siguientes gramáticas:

$\mathbf{G_1}$	$\mathbf{G_2}$
$S \to SAc$	$S \to BbS$
$\mid cd$	$\mid dAd$
$A \to aBS$	$A \to fAa$
$\mid aBd$	$\mid \varepsilon$
$B \to bc$	$B \to cA$
	$\mid A$

a) Comprueba si son LL(1). En caso de que no lo sean, propón una gramática LL(1) equivalente.

- G_1 no es $LL(1)$: $\begin{cases} \text{tiene reglas recursivas: } S \to SAc, S \to cd \\ \text{tiene reglas con prefijos comunes: } A \to aBS, A \to aBd \end{cases}$

  Gramática equivalente, G_1': $\left\{ \begin{array}{lll} S \to cdS' & A \to aBA' & B \to bc \\ S' \to AcS' & A' \to S & \\ S' \to \varepsilon & A' \to d & \end{array} \right\}$

  Para asegurar que G_1' es $LL(1)$, aplicar *test* $LL(1)$:
  - con S': $\text{FIRST}_1(AcS') = \{a\}, \text{FIRST}_1(\varepsilon) = \{\varepsilon\} \Rightarrow$ usar $\text{FOLLOW}_1(S') = \{\varepsilon, c\}, \{a\} \cap \{\varepsilon, c\} = \emptyset$
    Nota: $\text{FOLLOW}_1(S') = \text{FOLLOW}_1(S) = \{\varepsilon\} \cup \text{FOLLOW}_1(A') = \{\varepsilon\} \cup \text{FOLLOW}_1(A) = \{\varepsilon, c\}$
  - con A': $\text{FIRST}_1(S) \cap \text{FIRST}_1(d) = \{c\} \cap \{d\} = \emptyset$

  Todos son disjuntos $\Rightarrow$ G_1' **es** $LL(1)$

- G_2 no tiene problemas aparentes $\Rightarrow$ emplear el *test* $LL(1)$ [1]
  - con S : $\left\{ \begin{array}{l} \text{FIRST}_1(BbS) = \{c, f, b\} \\ \text{FIRST}_1(dAd) = \{d\} \end{array} \right\}$, $\text{FIRST}_1(BbS) \cap \text{FIRST}_1(dAd) = \emptyset$ (disjuntos)
  - con A : $\left\{ \begin{array}{l} \text{FIRST}_1(fAa) = \{f\} \\ \varepsilon \in \text{FIRST}_1(\varepsilon) = \{\varepsilon\} \Rightarrow \text{ usar } \text{FOLLOW}_1(A) = \{d, a, b\} \end{array} \right\}$,
    $\text{FIRST}_1(fAa) \cap \text{FOLLOW}_1(A) = \emptyset$ (disjuntos)
  - con B : $\left\{ \begin{array}{l} \text{FIRST}_1(cA) = \{c\} \\ \text{FIRST}_1(A) = \{f, \varepsilon\} \\ \varepsilon \in \text{FIRST}_1(A) \Rightarrow \text{ usar } \text{FOLLOW}_1(B) = \{b\} \end{array} \right\}$,
    $\text{FIRST}_1(cA) \cap \text{FIRST}_1(A) = \emptyset$ y $\text{FIRST}_1(cA) \cap \text{FOLLOW}_1(B) = \emptyset$ (disjuntos)

  Todos son disjuntos $\Rightarrow$ G_2 **es** $LL(1)$

b) Contruye la tabla de análisis LL(1) para G_2 (si no fuera LL(1), utiliza la gramática equivalente)

- Crear las entradas de la tabla:
  1) $S \to BbS$, $\text{FIRST}_1(BbS) = \{c, f, b\} \Rightarrow \left\{ \begin{array}{l} M[S,c] = (Bbs, 1) \\ M[S,f] = (Bbs, 1) \\ M[S,b] = (Bbs, 1) \end{array} \right\}$
  2) $S \to dAa$, $\text{FIRST}_1(aAd) = \{d\} \Rightarrow \{\ M[S,d] = (dAa, 2)\ \}$
  3) $A \to fAa$, $\text{FIRST}_1(fAa) = \{f\} \Rightarrow \{\ M[A,f] = (fAa, 3)\ \}$
  4) $A \to \varepsilon$, $\text{FIRST}_1(\varepsilon) = \{\varepsilon\} \Rightarrow$ usar $\text{FOLLOW}_1(A) = \{d, a, b\} \Rightarrow \left\{ \begin{array}{l} M[A,d] = (\varepsilon, 4) \\ M[A,a] = (\varepsilon, 4) \\ M[A,b] = (\varepsilon, 4) \end{array} \right\}$
  5) $B \to cA$, $\text{FIRST}_1(cA) = \{c\} \Rightarrow \{\ M[B,c] = (cA, 5)\ \}$
  6) $B \to A$, $\text{FIRST}_1(A) = \{f, \varepsilon\} \Rightarrow \left\{ \begin{array}{l} M[B,f] = (A, 6) \\ \varepsilon \in \text{FIRST}_1(A), \text{ usar } \text{FOLLOW}_1(B) = \{b\} \Rightarrow M[B,b] = (A, 6) \end{array} \right\}$

[1] Conjuntos FIRST_1 y FOLLOW_1:

	FIRST_1	FOLLOW_1
S	$\{d, c, f, b\}$	$\{\varepsilon\}$
A	$\{f, \varepsilon\}$	$\{d, a, b\}$
B	$\{c, f, \varepsilon\}$	$\{b\}$

- Tabla resultante:

	a	b	c	d	f	ε
S		$(BbS,1)$	$(BbS,1)$	$(dAd,2)$	$(BbS,1)$	
A	$(\varepsilon,4)$	$(\varepsilon,4)$		$(\varepsilon,4)$	$(fAa,3)$	
B		$(A,6)$	$(cA,5)$		$(A,6)$	
a	QUITAR					
b		QUITAR				
c			QUITAR			
d				QUITAR		
f					QUITAR	
$\$$						ACEPTAR

3. Dada la siguiente gramática:

$\mathbf{G_3}$

$$
\begin{array}{lll}
S \rightarrow Sbda & A \rightarrow bBBd & B \rightarrow Ba \\
\quad \mid Ad & \quad \mid a & \quad \mid S
\end{array}
$$

a) Construye su tabla de precedencia simple.

- **Relaciones $\doteq$:**
$$
\begin{cases}
S \rightarrow Sbda: & S \doteq b, b \doteq d, d \doteq a \\
S \rightarrow Ad: & A \doteq d \\
A \rightarrow bBBd: & b \doteq B, B \doteq B, B \doteq d \\
B \rightarrow Ba: & B \doteq a
\end{cases}
$$

- **Relaciones $\lessdot$:**
$$
\begin{cases}
A \rightarrow \mathbf{bB}Bd: & b \lessdot B, b \lessdot S, b \lessdot A, b \lessdot b, b \lessdot a \\
A \rightarrow b\mathbf{BB}d: & B \lessdot B, B \lessdot S, B \lessdot A, B \lessdot b, B \lessdot a \\
 & \text{las deriv. de } B \text{ empiezan por } \{B, S, A, b, a\}
\end{cases}
$$

- **Relaciones $\gtrdot$:**
$$
\begin{cases}
S \rightarrow \mathbf{Sb}da: & a \gtrdot b, d \gtrdot b \\
 & \text{las deriv. de } S \text{ acaban en } \{a, d\} \\
S \rightarrow \mathbf{Ad}: & d \gtrdot d, a \gtrdot d \\
 & \text{las deriv. de } A \text{ acaban en } \{d, a\} \\
A \rightarrow b\mathbf{BB}d: & a \gtrdot b, S \gtrdot b, d \gtrdot b, a \gtrdot a, S \gtrdot a, d \gtrdot a \\
 & \text{las deriv. de } B \text{ acaban en } \{a, S, d\} \\
 & \text{las deriv. de } B \text{ empiezan por los terminales } \{b, a\} \\
A \rightarrow bB\mathbf{Bd}: & a \gtrdot d, S \gtrdot d, d \gtrdot d \\
 & \text{las deriv. de } B \text{ acaban en } \{a, S, d\} \\
B \rightarrow \mathbf{Ba}: & a \gtrdot a, S \gtrdot a, d \gtrdot a \\
 & \text{las deriv. de } B \text{ acaban en } \{a, S, d\}
\end{cases}
$$

- **Relaciones con \$:**
$$
\begin{cases}
\$ \lessdot S, \$ \lessdot A, \$ \lessdot b, \$ \lessdot a \\
a \gtrdot \$, d \gtrdot \$
\end{cases}
$$

Tabla de precedencias:

	S	A	B	a	b	d	$\$$
S				$\gtrdot$	$\doteq, \gtrdot$	$\gtrdot$	
A						$\doteq$	
B	$\lessdot$	$\lessdot$	$\lessdot, \doteq$	$\lessdot, \doteq$	$\lessdot$	$\doteq$	
a				$\gtrdot$	$\gtrdot$	$\gtrdot$	$\gtrdot$
b	$\lessdot$	$\lessdot$	$\lessdot, \doteq$	$\lessdot$	$\lessdot$	$\doteq$	
d				$\doteq, \gtrdot$	$\gtrdot$	$\gtrdot$	$\gtrdot$
$\$$	$\lessdot$	$\lessdot$		$\lessdot$	$\lessdot$		